Extended Introduction to Computer Science CS1001.py

Module G:
Open Indexing and Cuckoo Hashing (for reference only)

Instructors: Elhanan Borenstein, Michal Kleinbort

Teaching Assistants: Noam Parzanchevsky,

Asaf Cassel, Shaked Dovrat, Omri Porat

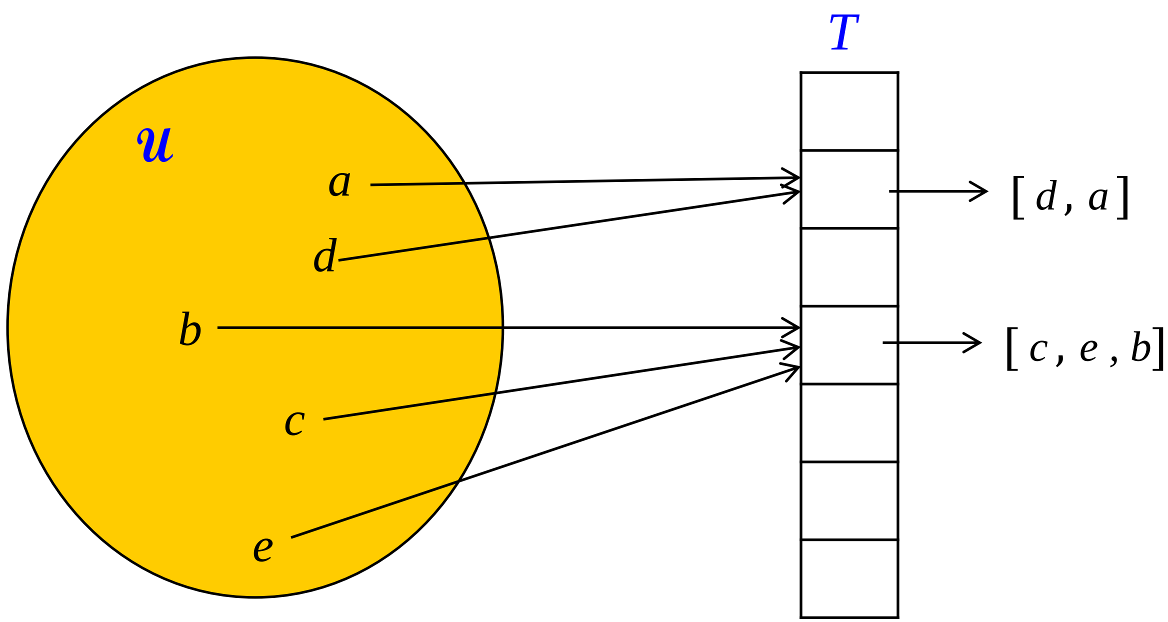
School of Computer Science
Tel-Aviv University

Spring Semester 2020-21

http://tau-cs1001-py.wikidot.com

Dealing with Collisions: Chaining Method

- Each cell in the table will contain a list (can be linked or not), with all the elements h maps to this cell



- How do we insert, search and delete elements?

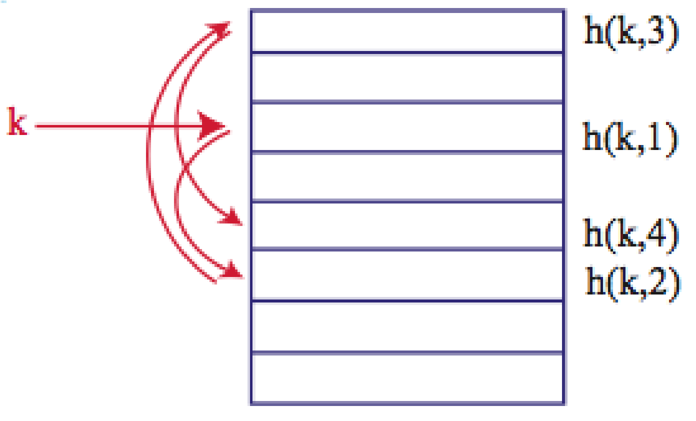
Two Approaches for Dealing with Collisions

1) Chaining – explained and implemented this ✓

2) Open addressing – we will briefly discuss it now

Two Approaches for Dealing with Collisions: (2) Open Addressing

- In open addressing, each slot in the hash table contains at most one item. This obviously implies that n cannot be larger than m.
- Furthermore, an item will typically not stay statically in the slot where it “tried'' to enter, or where it was placed initially. Instead, it may be moved a few times around.



- Open addressing is important in hardware applications where devices have many slots but each can only store one item (e.g. fast switches and high capacity routers). It is also used in python dictionaries and sets.
- There are many approaches to open addressing. We will describe a fairly recent one, termed cuckoo hashing (Pagh and Rodler, 2001).

Cuckoo Hashing: Motivation

- We saw that if $n \le m$, hashing with chaining guarantees that insertion, deletion, and find are carried out in expected time O(1) per operation, and with high probability (probability is over choices of inputs) O(log n/log log n) per operation. (The worst case time is O(n) per operation.)

- In certain scenarios (e.g. fast routers in large internet nodes) we want find to run with high probability in O(1) time. (The worst case time is still O(n) per operation.)

- Compare O(1) time with high probability to O(1) expected time of hashing with chaining.

- Cuckoo hashing is one way to achieves this, but there are two prices to pay:
  1) Instead of $n \le m$, we require $n \le 7m/8$, or $n \le 3m/4$, or $n \le m/2$, or even $n \le m/3$.
     - That is, we pay a price in terms of memory

  2) insert may take somewhat longer time.

Cuckoo Hashing

- Cuckoo hashing uses two distinct hash functions, h_1 and h_2 (improved versions use four, six, or eight, but the idea is the same).

- Each key, k , has two potential slots in the hash table, $h_1(k)$ and $h_2(k)$. If we search for k, all we have to do is look for it in these two locations (no chains here -- at most one item per slot).

- It is slightly more involved to insert a record whose key is k .

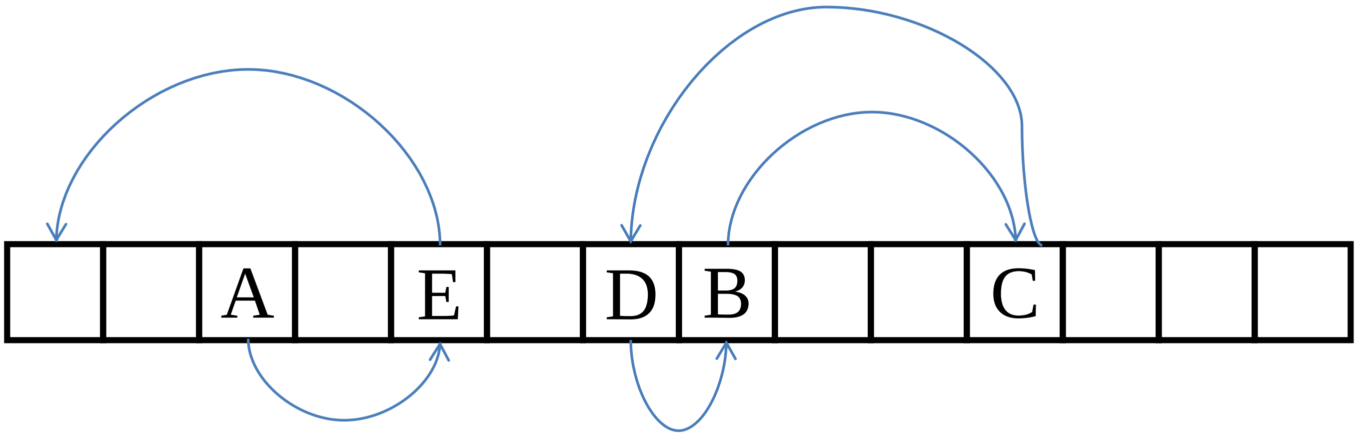
Cuckoo Hashing

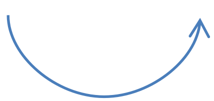
It is slightly more involved to insert a record whose key is k .

- If any of the two slots, $h_1(k)$ or $h_2(k)$ is empty, k is inserted there.
- If both slots are full, pick one of the two occupants, say x. Place k in x's current slot.
- Assume this was location $h_1(x)$. Place x in its other slot, $h_2(x)$.
- If that slot was empty, we are done.

- Otherwise, the slot is occupied by some y . Place this y in its other slot, potentially kicking its present occupant, etc.,etc., until we find an empty slot.

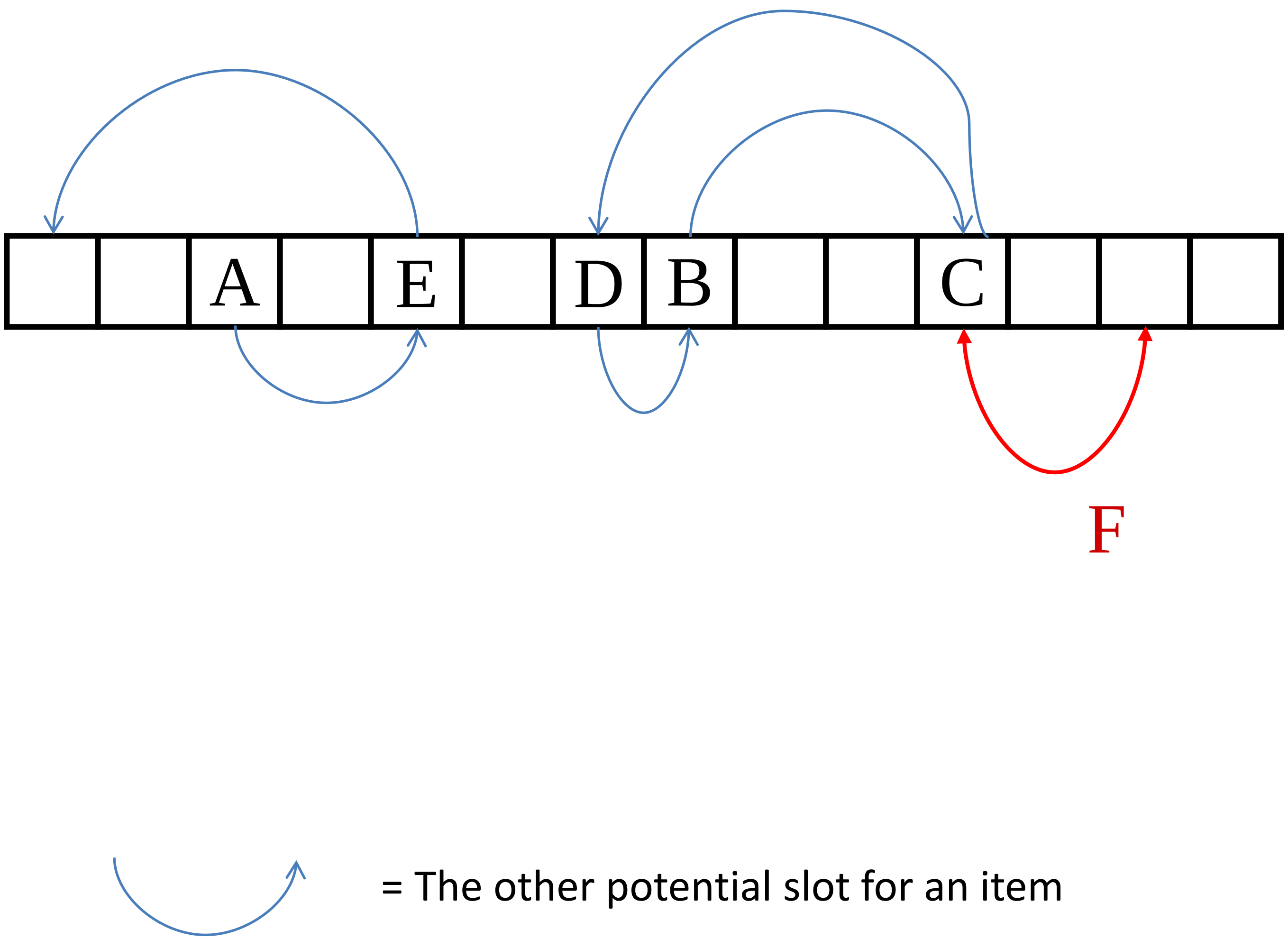
Cuckoo Hashing: Examples



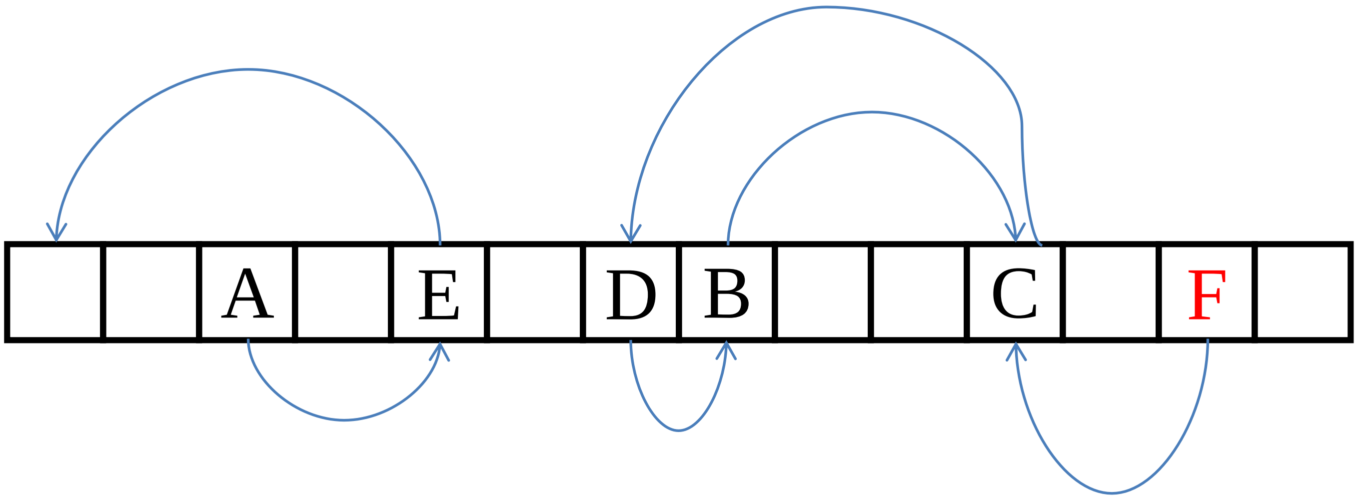


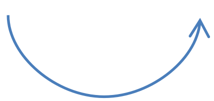
= The other potential slot for an item

Cuckoo Hashing: Examples



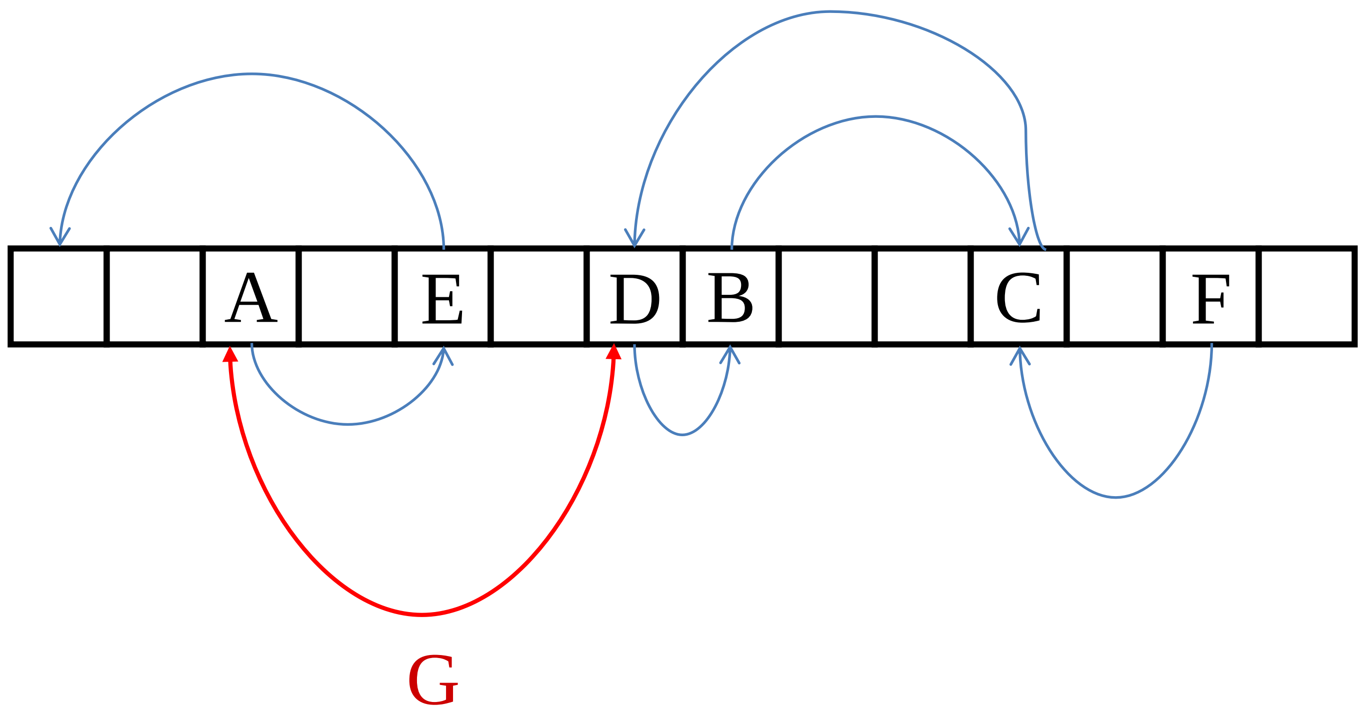
Cuckoo Hashing: Examples

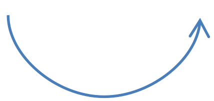




= The other potential slot for an item

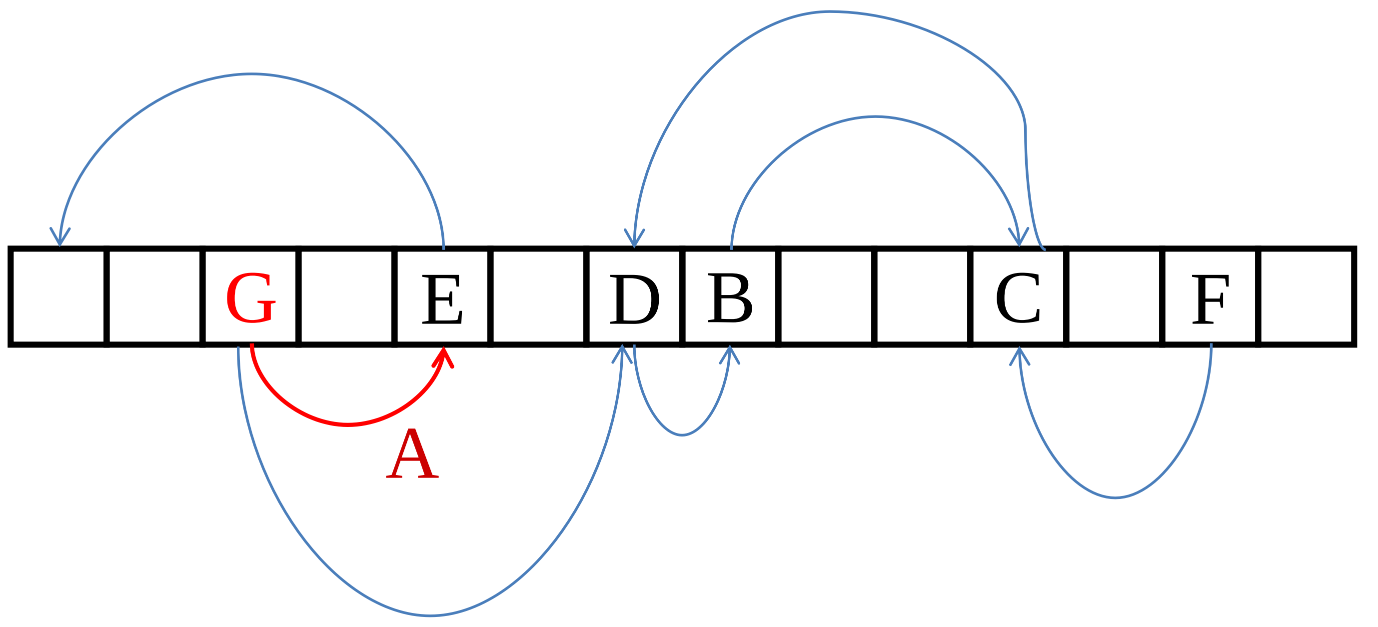
Cuckoo Hashing: Examples

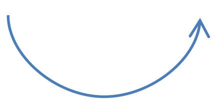




= The other potential slot for an item

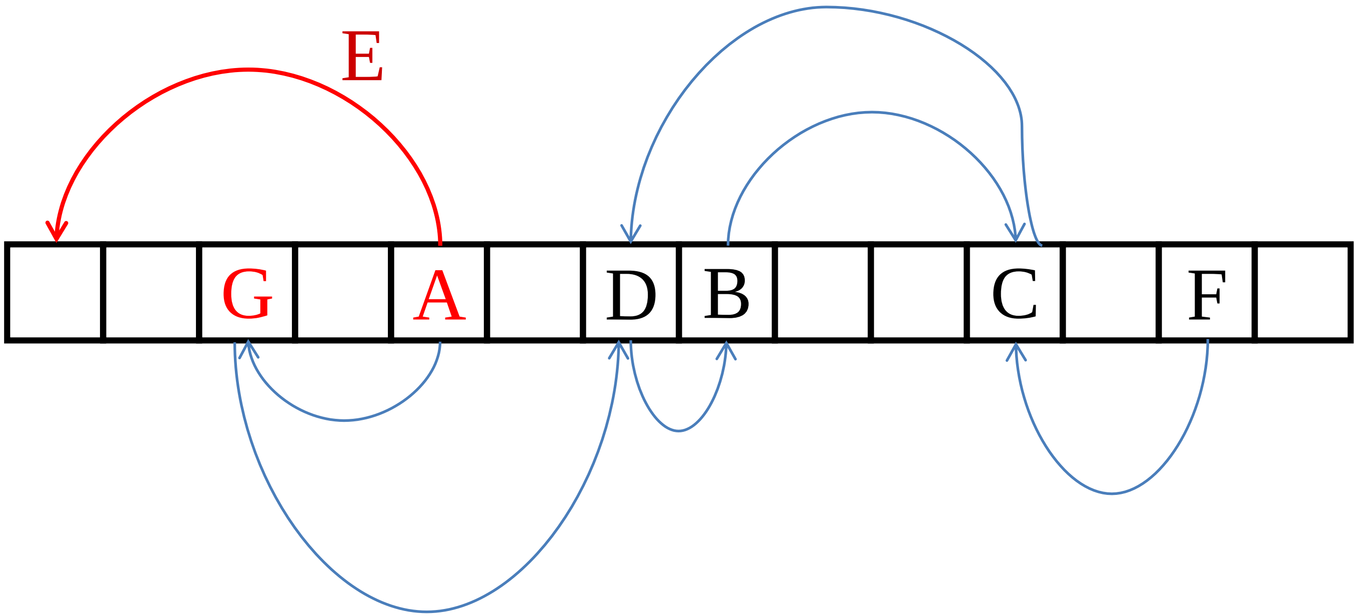
Cuckoo Hashing: Examples

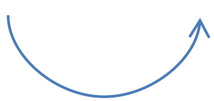




= The other potential slot for an item

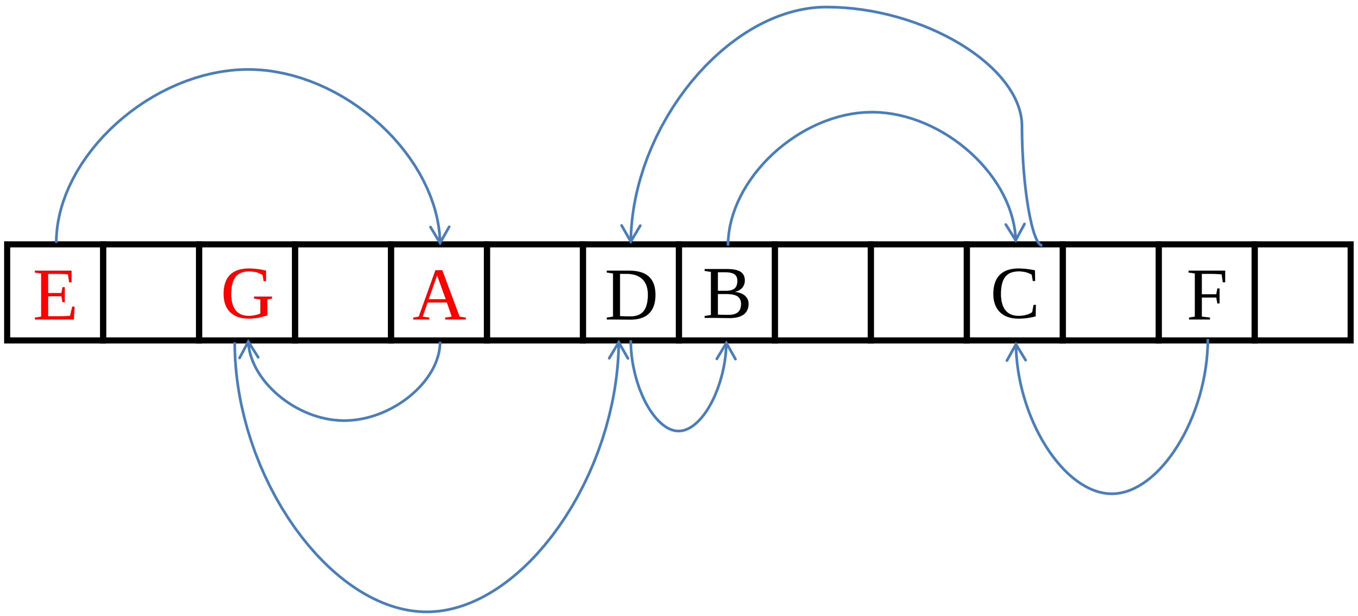
Cuckoo Hashing: Examples

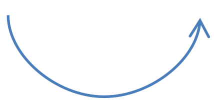




= The other potential slot for an item

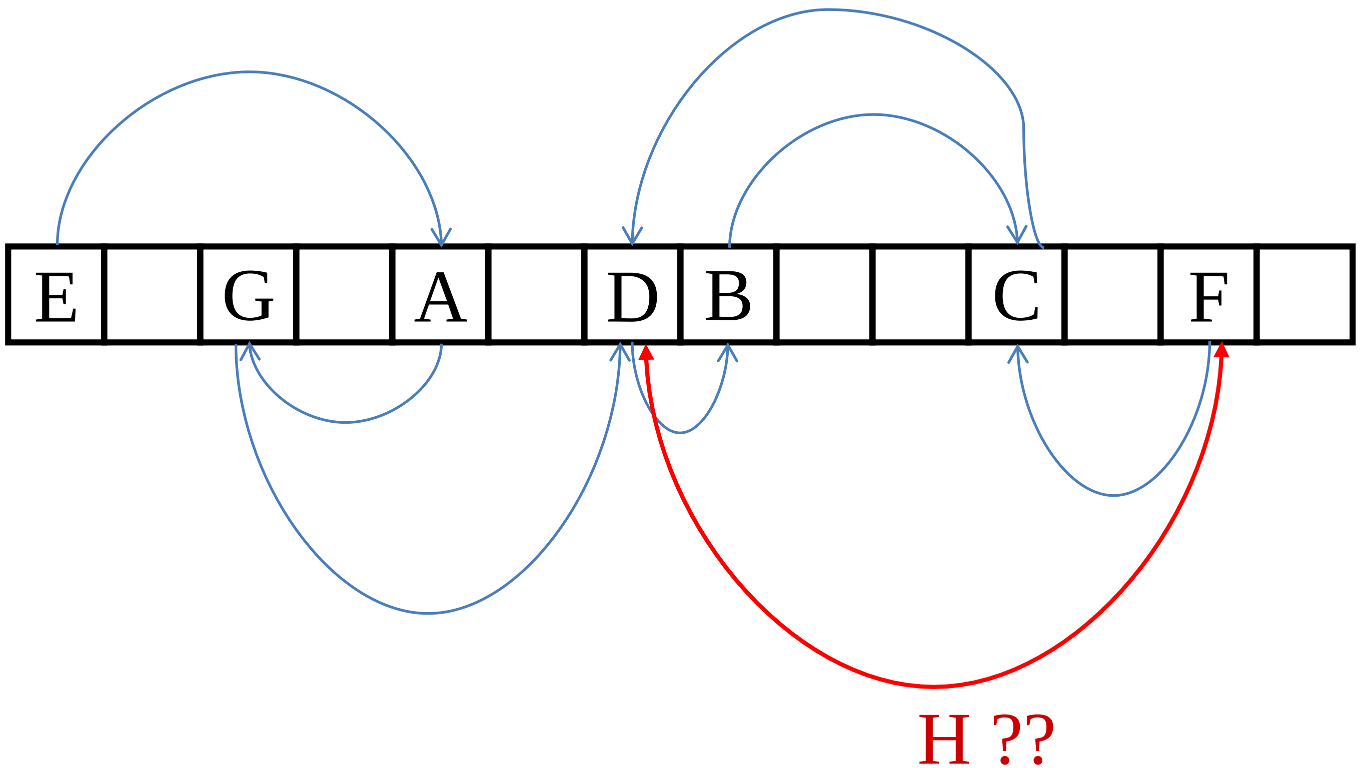
Cuckoo Hashing: Examples

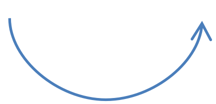




= The other potential slot for an item

Cuckoo Hashing: Examples




= The other potential slot for an item

Cuckoo Hashing - Deadlocks

- In the last example, we have reached a cycle, and we are in a non ending loop. This is called a deadlock.
- The union of the potential locations of 5 items (B, C, D, F, H) is just 4 slots.
- This obviously is very bad news for our cuckoo hashing.
- Notice that this is not a very likely event. With very high probability, the 10 potential locations ($10=5\cdot2$) will attain more than just 4 distinct values (which is why we got stuck in the last example).

Cuckoo Hashing – Solving Deadlocks

- Another possible problem is that there will be no cycle, but the path leading to the successful insertions will be very long.

- Fortunately, such unfortunate cases occur with very low probability when the load factor , i.e. n/m, is sufficiently low. The common recommendation for two hash functions , $h_1(\cdot), h_2(\cdot)$, is to have n/m < 1/2 . (More hash functions enable a higher load factor).

- <u>A theoretical solution</u>: In case of failure (or very long path), rehash using "fresh hash functions."

- <u>A more practical solution</u>: Maintain a very small excess zone (e.g. 32 excess slots for a hash table with m=10000 slots) and place items "causing trouble'' there. If regular search (applying $h_1(x)$, $h_2(x)$) fails, search the excess zone as well.

Cuckoo Hashing in the Real World

- The load factor has to be smaller than 1. Yet a small load factor, say $n/m < 1/2$, is a waste of memory.
- In high performance routers, for example, most operations (including the hashing) are done in silico, by the hardware. The critical resource is memory area within the chip. Low load factor means wasted area.
- Instead of just 2 hash functions, 4 to 8 hash functions are utilized. This allows to increase the load factor to $n/m = 3/4$ or even $n/m = 7/8$.
- Suppose we use 4 hash functions, $h_1()$, $h_2()$, $h_3()$, $h_4()$. Given an element, x , that we wish to insert, we first check if any of the four locations $h_1(x)$, $h_2(x)$, $h_3(x)$, $h_4(x)$ is free.

Cuckoo Hashing in the Real World, cont.

- If these 4 locations are all taken, let *a,b,c,d* be the four elements in the above mentioned locations, respectively.

- Look, for example, at *a*. If one of the other 3 locations among $h_1(a)$, $h_2(a)$, $h_3(a)$, $h_4(a)$ is free, we move *a* there, and put x in its place. If not, we do the same with respect to *b*, then *c*, then *d*.

- If all these are taken ($4+4\cdot3=16$ different locations, typically), we go one more level down this search tree ($12\cdot3 = 36$ additional locations, typically).

- If all these are taken, we give up on x and put it in the garbage bin (“excess zone“ table).

- With very high probability, the small excess zone does not fill up. After removing elements from the table, we could try re-inserting such x to the hash table.

Designing Distinct Hash Functions

- Recall that the goal of designing a hash functions is that they map most sets of keys such that the maximal number of collisions is small.

- When having more than one hash functions, we have the additional goal that the different functions map same keys approximately independently. In Python, we could try variants of good ole hash.

For example:

```
def hash0(x):
    return hash("0" + str(x))
def hash1(x):
    return hash("1" + str(x))
def hash2(x):
    return hash(str(x) + "2")
def hash3(x):
    return hash(str(x) + "3")
```

Designing Distinct Hash Functions

A reminder concerning str (mapping objects to representing strings):

```
>>> [str(i) for i in range(10,20)]
['10', '11', '12', '13', '14', '15', '16', '17', '18', '19']
>>> str(2.2)
'2.2'
>>> str("2.2")
'2.2'
```

And now applying the four functions on a small domain:

```
>>> for f in (hash0,hash1,hash2,hash3):
        print([f(i) %23 for i in range(10,20)])
[ 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]
[ 3, 2, 5, 4, 22, 21, 1, 0, 11, 10]
[12, 5, 17, 10, 16, 9, 7, 0, 10, 22]
[13, 4, 18, 9, 17, 8, 8, 22, 11, 21]
```

Random? Independent? Mixing well? You be the judges.